Rossmoyne Senior High School

Semester Two Examination, 2019

Question/Answer booklet

MATHEMATICS APPLICATIONS UNITS 3 AND 4

Section One: Calculator-free

SOLUTIONS

Student number: In figures | | | | | | | | |

In words _____

Your name _____

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answer to the specific question asked and to follow any instructions that are specified to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

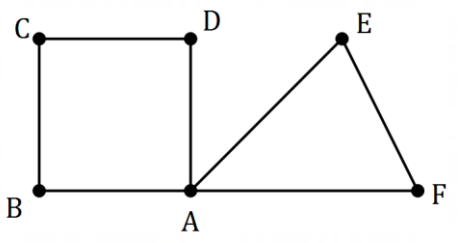
Section One: Calculator-free **35% (52 Marks)**

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 **(4 marks)**

Graph G is shown below.



(a) State the length of the shortest cycle in G and list the vertices in this cycle. (2 marks)

Solution
Length is 3. Vertices in cycle are A, E, F.
Specific behaviours
✓ length ✓ list of vertices (can repeat start/end vertex)

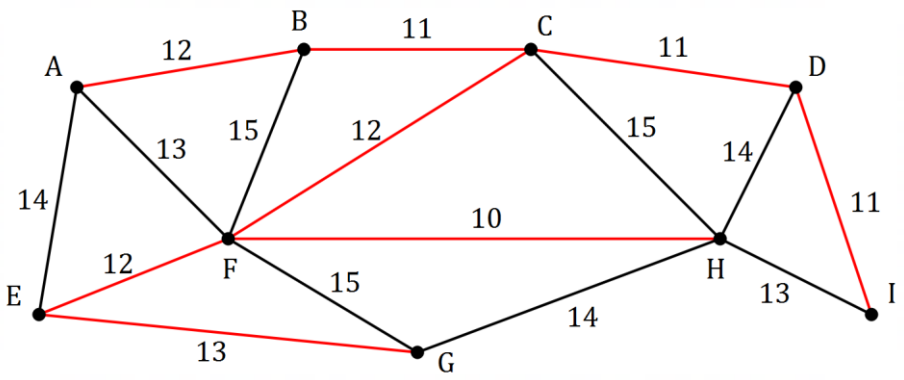
(b) State the length of the longest Hamiltonian path in G and list all possible starting vertices for this path. (2 marks)

Solution
Length is 5. Can start at B, D, E or F.
Specific behaviours
✓ length ✓ lists all possible vertices

See next page

Question 2 **(6 marks)**

The weights on the graph below are the costs, in hundreds of dollars, to connect adjacent offices (represented by the vertices) to a new IT system.



(a) Cleary indicate the minimum spanning tree on the gra (2 marks)

Solution
See diagram
Specific behaviours
✓ any tree
✓ correct tree

(b) Determine the cost of connecting the offices to the new IT system using the minimum spanning tree. (2 marks)

F.T

Solution
$12 + 11 + 12 + 12 + 10 + 11 + 11 + 13 = 92$ $\text{Cost} = 92 \times 100 = \$9\,200$
Specific behaviours
✓ sum of edges shown on graph
✓ correct cost in hundreds

(c) An IT consultant recommends that the new system must include a connection between office C and office F, and between office C and office H. Determine the minimum cost of connecting all the offices using a spanning tree that includes these two edges. (2 marks)

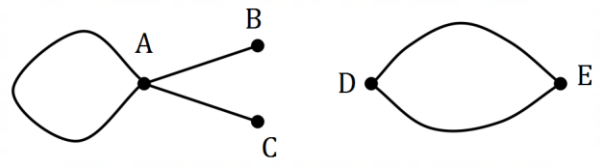
F.T

Solution
CF makes no difference but CH will increase sum of edge weights by $15 - 10 = 5$. So new minimum cost will be $9200 + 500 = \$9\,700$.
Specific behaviours
✓ indicates correct method
✓ correct minimum cost

See next page

Question 3 **(7 marks)**

(a) Graph G_1 below has 5 vertices.



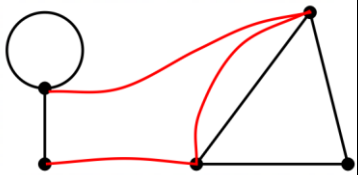
(i) Construct an adjacency matrix from G_1. (2 marks)

Solution
$\begin{array}{c} \\ A \\ B \\ C \\ D \\ \text{E} \end{array}\begin{array}{c} \begin{array}{ccccc} A & B & C & D & \text{E} \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 2 & 0 \end{bmatrix} \end{array}$
Specific behaviours
✓ at least 3 rows correct
✓ all rows correct

(ii) Give two reasons that G_1 is not simple. (2 marks)

Solution
- contains a loop - contains multiple edges
Specific behaviours
✓ loops ✓ multiple edges

(b) Graph G_2 below has 5 edges.



Solution (i)
See graph (example solution)
Specific behaviours
✓ Eulerian trail (even vertices) ✓ Hamiltonian cycle (Penalise if adds > 3 edges)

(i) Without adding any more vertices, add tree edges to G_2 so that it is Eulerian and Hamiltonian. (2 marks)

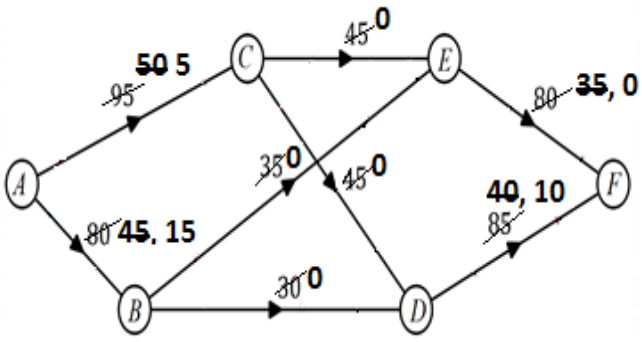
(ii) State the difference in length of the Eulerian trail and Hamiltonian cycle in the modified G_2. (1 mark)

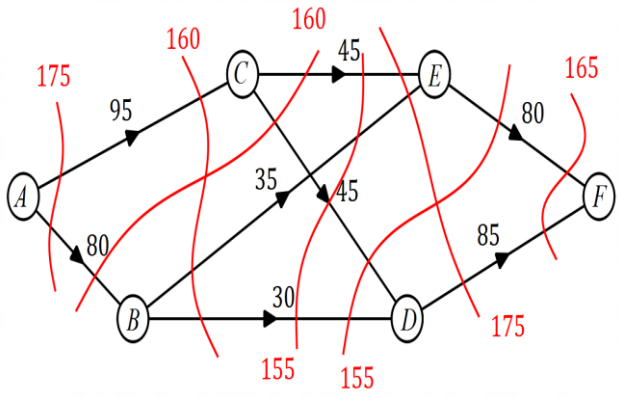
Solution
Difference $= 8 - 5 = 3$ edges
Specific behaviours
✓ correct difference

See next page

Question 4 **(7 marks)**

The digraph below shows the possible routes that a car can take to reach freeway entry F after they leave carpark A. The weights on each edge represent the maximum number of cars that can travel between adjacent intersections (vertices) every minute.





(a) Determine the maximum number of cars that can travel from A to F every minute. (3 marks)

Solution
$ACEF = 45$ $ACDF = 45$ $ABEF = 35$ $ABDF = 30$ Max flow $= 155$ cars per minute
Specific behaviours
✓ systematic listing of flows on diagram ✓ at least 3 correct paths ✓ correct maximum flow

Alternative Solution
See diagram - shows at least five cuts and states minimum cut is 155 cars per minute.
Specific behaviours
✓ systematically shows cuts ✓ at least 5 correct cuts ✓ correct maximum flow

(b) Determine, with justification, the maximum increase, if any, in the flow of cars every minute from A to F that could be achieved by adding a new route

(i) from A to E that can carry up to 25 cars per minute. (2 marks)

F.T

Solution
No increase. Edge EF is already at maximum capacity and so no additional flow from AE can be accommodated.
Specific behaviours
✓ states no increase ✓ correct explanation

(ii) from B to F that can carry up to 35 cars per minute. (2 marks)

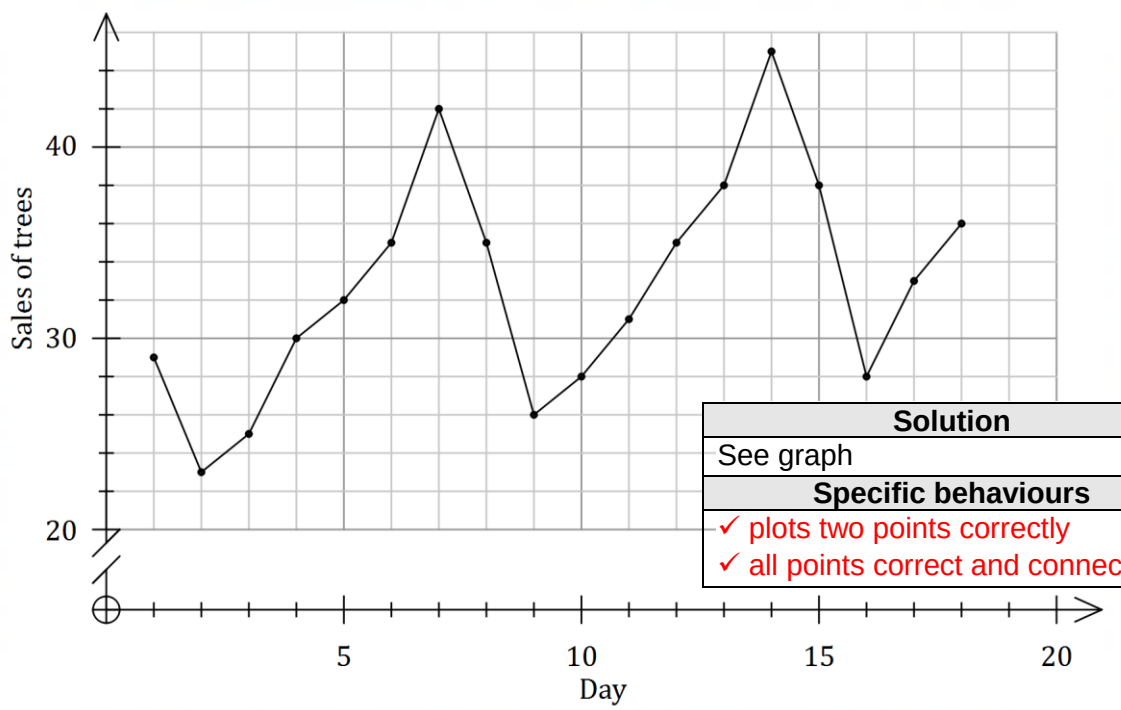
F.T

Solution
An increase of 15 cars per minute. Edge AB has spare capacity of 15 which could then flow through the new edge BF.
Specific behaviours
✓ states increase ✓ correct explanation

See next page

Question 5 **(6 marks)**

The time series plot below shows the number of trees sold by a garden centre over 15 consecutive days at the start of September. Day 1 was a Sunday.



Solution
See graph
Specific behaviours
✓ plots two points correctly
✓ all points correct and connected

(a) Determine the day of the week that day 12 represents. (1 mark)

Solution
Thursday
Specific behaviours
✓ Correct day

(b) On days 16, 17 and 18 the centre sold 28, 33 and 36 trees respectively. Use this data to complete the time series plot. (2 marks)

(c) Describe the trend and seasonality displayed in the time series plot. (3 marks)

Solution
There is an increasing trend. There is a weekly season, with a pattern of increasing and decreasing sales, peaking on Saturday from a low on Mondays.
Specific behaviours
✓ describes trend as increasing or positive ✓ describes seasonality as 7 points or weekly ✓ describes peaks and lows

See next page

Question 6 **(7 marks)**

The quarterly sales of a drilling machine are shown in the table below.

Year	2017			2018				2019
Quarter	2	3	4	1	2	3	4	1
Sales	24	22	27	30	23	18	24	–

(a) Calculate the 4-point centred moving average for quarter 1 of 2018. (2 marks)

Solution
$22 \div 2 + 27 + 30 + 23 + 18 \div 2 = 100$ $100 \div 4 = 25$
Specific behaviours
✓ indicates use of correct method ✓ correct average

(b) Determine the sales for quarter 1 of 2019 given that the 3-point moving average for quarter 4 of 2018 was 25. (2 marks)

Solution
$18 + 24 + x = 3 \times 25$ $x = 33$ There were 33 sales
Specific behaviours
✓ indicates use of correct method ✓ correct sales

(c) Moving averages are often calculated to smooth out time series data. Explain why this is useful. (1 mark)

Solution
To identify the underlying trend.
Specific behaviours
✓ valid reason

(d) The figures in the table clearly indicate that for one of the seasons, the deseasonalised sales will be higher than the actual sales. Name this season and explain your answer. (2 marks)

Solution
In quarter 3, since sales are at their lowest each year during this season.
Specific behaviours
✓ correct quarter ✓ valid explanation

See next page

Question 7 **(7 marks)**

A company sells water coolers and each day allocates staff to call businesses and make sales in its four sales regions. One staff member is assigned to one region for the whole day.

One day, only three sales staff turn up to work and the manager must decide how to allocate them to maximise the total number of sales made. The table below shows the expected number of sales each staff member will make in each region.

		Sales Region			
		V	W	X	Y
Staff	Amy	35	40	37	33
	Beth	24	29	28	25
	Corey	32	33	29	28

(a) Show use of the Hungarian algorithm to determine the optimum allocation of staff to regions in order to maximise sales on this day. (6 marks)

Solution

$$\begin{matrix} 5 & 0 & 3 & 7 \\ 16 & 11 & 12 & 15 \\ 8 & 7 & 11 & 12 \\ 0 & 0 & 0 & 0 \end{matrix}$$

$$\begin{matrix} 5 & 0 & 3 & 7 \\ 5 & 0 & 1 & 4 \\ 1 & 0 & 4 & 5 \\ 0 & 0 & 0 & 0 \end{matrix}$$

$$\begin{matrix} 4 & \mathbf{0} & 2 & 6 \\ 4 & 0 & \mathbf{0} & 3 \\ \mathbf{0} & 0 & 3 & 4 \\ 0 & 1 & 0 & \mathbf{0} \end{matrix}$$

Allocation: Amy to W, Beth to X and Corey to V.

Specific behaviours

✓ subtracts all elements from largest number
✓ adds row of zeros
✓ reduces rows so all contain a zero
✓ covers zeroes, adjusts uncovered and twice covered
✓ covers zeroes with four lines
✓ states assignment

(b) State the total number of sales on this day using your allocation from (a). (1 mark)

Solution

$40 + 28 + 32 = 100$ sales

Specific behaviours

✓ correct number using allocation

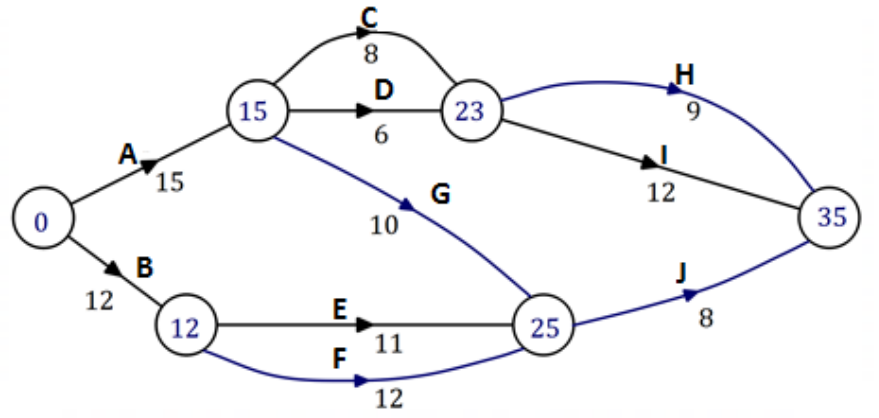
See next page

Question 8 **(8 marks)**

A project consists of activities A_1 to A_{10}. The duration and immediate predecessors for each activity are shown in the table below.

Activity	A	B	C	D	E	F	G	H	I	J
Duration (minutes)	15	12	8	6	11	12	10	9	12	8
Immediate predecessors	–	–	A	A	B	B	A	C, D	C, D	E, F, G

(a) Complete the network below to represent the durations and interdependencies of all the activities in the project. (2 marks)



Solution
See diagram
Specific behaviours
✓ all weights correct
✓ all edges + labels correct

(b) Determine the earliest starting time for J. (1 mark)

Solution
After 25 minutes
Specific behaviours
✓ correct time in minutes

(c) List, in order, the activities that lie on the critical path and state the minimum completion time for the project. (2 marks)

F.T

Solution
Critical path **A,C,I** and MCT $= 35$ minutes
Specific behaviours
✓ correct critical path
✓ correct MCT

(d) Determine the latest starting time for E. (1 mark)

F.T

Solution
After 16 minutes
Specific behaviours
✓ correct time ($MCT - 19$)

(e) If the duration of E was increased by 5 minutes, what effect, if any, would this have on the critical path and minimum completion time? (2 marks)

F.T

Solution
New CP is **B,E,J** and new MCT $= 36$ minutes
Specific behaviours
✓ states new CP
✓ states new MCT

End of questions

Supplementary page

Question number: _____